Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced Subsidiary

Monday 30 January 2012 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation or integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Further Pure Mathematics FP1), the paper reference (6667), your surname, initials and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 9 questions on this paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P40086A

1. Given that $z_1 = 1 - \text{i}$,

(*a*) find $\arg(z_1)$.

(2)

Given also that $z_2 = 3 + 4\text{i}$, find, in the form $a + \text{i}b$, $a, b \in \mathbb{R}$,

(*b*) $z_1 z_2$,

(2)

(*c*) $\dfrac{z_2}{z_1}$.

(3)

In part (*b*) and part (*c*) you must show all your working clearly.

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2. (*a*) Show that $f(x) = x^4 + x - 1$ has a real root α in the interval [0.5, 1.0].

(2)

(*b*) Starting with the interval [0.5, 1.0], use interval bisection twice to find an interval of width 0.125 which contains α.

(3)

(*c*) Taking 0.75 as a first approximation, apply the Newton Raphson process twice to $f(x)$ to obtain an approximate value of α. Give your answer to 3 decimal places.

(5)

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3. A parabola C has cartesian equation $y^2 = 16x$. The point $P(4t^2, 8t)$ is a general point on C.

(*a*) Write down the coordinates of the focus F and the equation of the directrix of C.

(3)

(*b*) Show that the equation of the normal to C at P is $y + tx = 8t + 4t^3$.

(5)

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4. A right angled triangle T has vertices $A(1, 1)$, $B(2, 1)$ and $C(2, 4)$. When T is transformed by the matrix $\mathbf{P} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, the image is T'.

(*a*) Find the coordinates of the vertices of T'.

(2)

(*b*) Describe fully the transformation represented by **P**.

(2)

The matrices $\mathbf{Q} = \begin{pmatrix} 4 & -2 \\ 3 & -1 \end{pmatrix}$ and $\mathbf{R} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ represent two transformations. When T is transformed by the matrix **QR**, the image is T''.

(*c*) Find **QR**.

(2)

(*d*) Find the determinant of **QR**.

(2)

(*e*) Using your answer to part (*d*), find the area of T''.

(3)

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5. The roots of the equation

$$z^3 - 8z^2 + 22z - 20 = 0$$

are z_1, z_2 and z_3 .

(*a*) Given that $z_1 = 3 + \text{i}$, find z_2 and z_3.

(4)

(*b*) Show, on a single Argand diagram, the points representing z_1, z_2 and z_3.

(2)

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6. (*a*) Prove by induction

$$\sum_{r=1}^{n} r^3 = \frac{1}{4} n^2(n + 1)^2.$$

(5)

(*b*) Using the result in part (*a*), show that

$$\sum_{r=1}^{n} (r^3 - 2) = \frac{1}{4} n(n^3 + 2n^2 + n - 8).$$

(3)

(*c*) Calculate the exact value of $\sum_{r=20}^{50} (r^3 - 2)$.

(3)

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7. A sequence can be described by the recurrence formula

$$u_{n+1} = 2u_n + 1, \qquad n \geq 1, \quad u_1 = 1.$$

(*a*) Find u_2 and u_3.

(2)

(*b*) Prove by induction that $u_n = 2^n - 1$.

(5)

8.
$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}.$$

(*a*) Show that **A** is non-singular.

(2)

(*b*) Find **B** such that $\mathbf{BA}^2 = \mathbf{A}$.

(4)

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9. The rectangular hyperbola *H* has cartesian equation $xy = 9$.

The points $P\left(3p, \frac{3}{p}\right)$ and $Q\left(3q, \frac{3}{q}\right)$ lie on *H*, where $p \neq \pm q$.

(*a*) Show that the equation of the tangent at *P* is $x + p^2y = 6p$.

(4)

(*b*) Write down the equation of the tangent at *Q*.

(1)

The tangent at the point *P* and the tangent at the point *Q* intersect at *R*.

(*c*) Find, as single fractions in their simplest form, the coordinates of *R* in terms of *p* and *q*.

(4)

TOTAL FOR PAPER: 75 MARKS

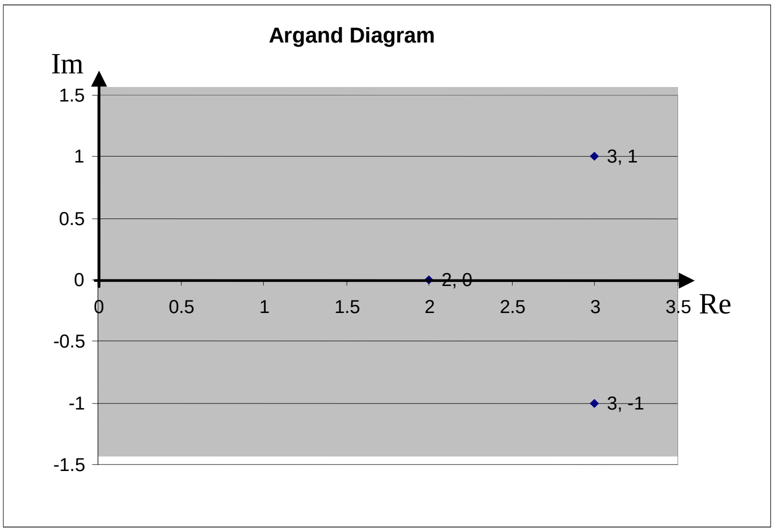
END

Question Number	Scheme	Notes	Marks
1(a)	$\arg z_1 = -\arctan(1)$	$-\arctan(1)$ or $\arctan(1)$ or $\arctan(-1)$	M1
	$= -\dfrac{\pi}{4}$	or -45 or awrt -0.785 (oe e.g $\dfrac{7\pi}{4}$)	A1
	Correct answer only 2/2		**(2)**
(b)	$z_1 z_2 = (1-i)(3+4i) = 3-3i+4i-4i^2$	At least 3 correct terms (Unsimplified)	M1
	$= 7 + \mathrm{i}$	cao	A1
			(2)
(c)	$\dfrac{z_2}{z_1} = \dfrac{(3+4\mathrm{i})}{(1-\mathrm{i})} = \dfrac{(3+4\mathrm{i}).(1+\mathrm{i})}{(1-\mathrm{i}).(1+\mathrm{i})}$	Multiply top and bottom by (1 + i)	M1
	$= \dfrac{(3+4\mathrm{i}).(1+\mathrm{i})}{2}$	$(1+\mathrm{i})(1-\mathrm{i}) = 2$	A1
	$= -\dfrac{1}{2} + \dfrac{7}{2}\mathrm{i}$	or $\dfrac{-1+7\mathrm{i}}{2}$	A1 **(3)**
	Correct answers only in (b) and (c) scores no marks		**(7 marks)**

Question Number	Scheme	Notes	Marks
2	$\mathrm{f}(x) = x^4 + x - 1$		
(a)	f(0.5) = -0.4375 $(-\frac{7}{16})$ f(1) = 1	Either any one of f(0.5) = awrt -0.4 or f(1) = 1	M1
	Sign change (positive, negative) (and $\mathrm{f}(x)$ is continuous) therefore (a **root)** α is between $x = 0.5$ and $x = 1.0$	f(0.5) = awrt -0.4 **and** f(1) = 1, sign change and conclusion	A1
			(2)
(b)	f(0.75) = 0.06640625$(\frac{17}{256})$	Attempt f(0.75)	M1
	f(0.625) = -0.222412109375$(-\frac{911}{4096})$	f(0.75) = awrt 0.07 **and** f(0.625) = awrt -0.2	A1
	$0.625 \leqslant \alpha \leqslant 0.75$	$0.625 \leqslant \alpha \leqslant 0.75$ or $0.625 < \alpha < 0.75$ or $[0.625, 0.75]$ or $(0.625, 0.75)$. or equivalent in words.	A1
	In (b) there is no credit for linear interpolation and a correct answer with no working scores no marks.		**(3)**
(c)	$\mathrm{f}'(x) = 4x^3 + 1$	Correct derivative (May be implied later by e.g. $4(0.75)^3 + 1$)	B1
	$x_1 = 0.75$		
	$x_2 = 0.75 - \frac{f(0.75)}{f'(0.75)} = 0.75 - \frac{0.06640625}{2.6875\left(43/16\right)}$	Attempt Newton-Raphson	M1
	$x_2 = 0.72529(06976...) = \frac{499}{688}$	Correct first application – a correct numerical expression e.g. $0.75 - \frac{17/256}{43/16}$ or awrt 0.725 (may be implied)	A1
	$x_3 = 0.724493\left(\frac{499}{688} - \frac{0.002015718978}{2.562146811}\right)$	Awrt 0.724	A1
	$(\alpha) = 0.724$	cao	A1
	A final answer of 0.724 with evidence of NR applied twice with no incorrect work should score 5/5		**(5)**
			(10 marks)

Question Number	Scheme	Notes	Marks
3(a)	Focus $(4, 0)$		B1
	Directrix $x+4=0$	x + "4" = 0 or x = - "4"	M1
		x + 4 = 0 or x = - 4	A1
			(3)
(b)	$y=4x^{\frac{1}{2}} \Rightarrow \frac{dy}{dx}=2x^{-\frac{1}{2}}$ $y^2=16x \Rightarrow 2y\frac{dy}{dx}=16$ or $\frac{dy}{dx}=\frac{dy}{dt}.\frac{dt}{dx}=8.\frac{1}{8t}$	$\frac{dy}{dx}=k\,x^{-\frac{1}{2}}$ $ky\frac{dy}{dx}=c$ their $\frac{dy}{dt}\times\left(\frac{1}{\text{their }\frac{dx}{dt}}\right)$	M1
	$\frac{dy}{dx}=2x^{-\frac{1}{2}}$ *or* $2y\frac{dy}{dx}=16$ *or* $\frac{dy}{dx}=8.\frac{1}{8t}$	Correct differentiation	A1
	At P, gradient of normal = $-t$	Correct normal gradient with no errors seen.	A1
	$y-8t=-t(x-4t^2)$	Applies $y-8t$ = their $m_N\left(x-4t^2\right)$ or $y=\left(\text{their } m_N\right)x+c$ using $x=4t^2$ *and* $y=8t$ in an attempt to find c. **Their m_N must be different from their m_T and must be a function of *t*.**	M1
	$y+tx=8t+4t^3$ *	cso **given answer**	A1
	Special case – if the correct gradient is <u>quoted</u> could score M0A0A0M1A1		**(5)**
			(8 marks)

Question Number	Scheme	Notes	Marks
4(a)	$\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\begin{pmatrix}1 & 2 & 2\\1 & 1 & 4\end{pmatrix}=\begin{pmatrix}1 & 1 & 4\\1 & 2 & 2\end{pmatrix}$	Attempt to multiply the right way round with at least 4 correct elements	M1
	T' has coordinates (1,1), (1,2) and (4,2) or $\begin{pmatrix}1\\1\end{pmatrix},\begin{pmatrix}1\\2\end{pmatrix},\begin{pmatrix}4\\2\end{pmatrix}$ **NOT just** $\begin{pmatrix}1 & 1 & 4\\1 & 2 & 2\end{pmatrix}$	Correct coordinates or vectors	A1
			(2)
(b)	**Reflection** in the line $\boldsymbol{y = x}$	Reflection	B1
		$y = x$	B1
	Allow 'in the axis' 'about the line' $y = x$ etc. Provided both features are mentioned ignore any reference to the origin unless there is a clear contradiction.		
			(2)
(c)	$\mathbf{QR}=\begin{pmatrix}4 & -2\\3 & -1\end{pmatrix}\begin{pmatrix}1 & 2\\3 & 4\end{pmatrix}=\begin{pmatrix}-2 & 0\\0 & 2\end{pmatrix}$	2 correct elements	M1
		Correct matrix	A1
	Note that $\mathbf{RQ}=\begin{pmatrix}1 & 2\\3 & 4\end{pmatrix}\begin{pmatrix}4 & -2\\3 & -1\end{pmatrix}=\begin{pmatrix}10 & -4\\24 & -10\end{pmatrix}$ **scores M0A0 in (c) but allow all the marks in (d) and (e)**		
			(2)
(d)	$\det(\mathbf{QR})=-2\times2-0=-4$	"-2"x"2" – "0"x"0"	M1
		-4	A1
	Answer only scores 2/2 $\dfrac{1}{\det(\mathbf{QR})}$ **scores M0**		**(2)**
(e)	Area of $T = \frac{1}{2}\times1\times3=\frac{3}{2}$	Correct area for T	B1
	Area of $T''=\frac{3}{2}\times4=6$	Attempt at "$\frac{3}{2}$"$\times\pm$"4"	M1
		6 or follow through their det(QR) x Their triangle area provided area > 0	A1ft
			(3)
			(11 marks)

Question Number	Scheme	Notes	Marks
5(a)	$(z_2) = 3 - i$		B1
	$(z-(3+i))(z-(3-i)) = z^2 - 6z + 10$	Attempt to expand $(z-(3+i))(z-(3-i))$ or any valid method to establish the quadratic factor e.g. $z = 3 \pm i \Rightarrow z - 3 = \pm i \Rightarrow z^2 - 6z + 9 = -1$ $z = 3 \pm \sqrt{-1} = \dfrac{6 \pm \sqrt{-4}}{2} \Rightarrow b = -6, c = 10$ Sum of roots 6, product of roots 10 $\therefore z^2 - 6z + 10$	M1
	$(z^2 - 6z + 10)(z - 2) = 0$	Attempt at linear factor with their *cd* in $(z^2 + az + c)(z + d) = \pm 20$ Or $(z^2 - 6z + 10)(z + a) \Rightarrow 10a = -20$ Or attempts f(2)	M1
	$(z_3) = 2$		A1
	Showing that $f(2) = 0$ **is equivalent to scoring both M's so it is possible to gain all 4 marks quite easily e.g.** $z_2 = 3 - i$ **B1, shows** $f(2) = 0$ **M2,** $z_3 = 2$ **A1.** **Answers only can score 4/4**		**(4)**
5(b)	 First B1 for plotting (3, 1) and (3, -1) correctly with an indication of scale or labelled with coordinates (allow points/lines/crosses/vectors etc.) Allow *i*/-*i* for 1/-1 marked on imaginary axis. Second B1 for plotting (2, 0) correctly relative to the conjugate pair with an indication of scale or labelled with coordinates or just 2		B1 B1 **(2)** **(6 marks)**

Question Number	Scheme	Notes	Marks
6(a)	$n=1, \text{LHS}=1^3=1, \text{RHS}=\frac{1}{4}\times1^2\times2^2=1$	Shows **both** LHS = 1 **and** RHS = 1	B1
	Assume true for n = k		
	When n = k + 1 $\sum_{r=1}^{k+1} r^3=\frac{1}{4}k^2(k+1)^2+(k+1)^3$	Adds $(k+1)^3$ to the given result	M1
	$=\frac{1}{4}(k+1)^2[k^2+4(k+1)]$	Attempt to factorise out $\frac{1}{4}(k+1)^2$	dM1
		Correct expression with $\frac{1}{4}(k+1)^2$ factorised out.	A1
	$=\frac{1}{4}(k+1)^2(k+2)^2$ Must see 4 things: <u>true for n = 1</u>, <u>assumption true for n = k</u>, <u>said true for n = k + 1</u> and therefore <u>true for all n</u>	Fully complete proof with no errors and comment. All the previous marks must have been scored.	A1cso
	See extra notes for alternative approaches		(5)
(b)	$\sum(r^3-2)=\sum r^3-\sum 2$	Attempt two sums	M1
	$\sum r^3-\sum 2n$ **is M0**		
	$=\frac{1}{4}n^2(n+1)^2-2n$	Correct expression	A1
	$=\frac{n}{4}(n^3+2n^2+n-8)$ *	Completion to printed answer with no errors seen.	A1
			(3)
(c)	$\sum_{r=20}^{r=50}(r^3-2)=\frac{50}{4}\times130042-\frac{19}{4}\times7592$ $(=1625525-36062)$	Attempt $S_{50}-S_{20}$ or $S_{50}-S_{19}$ and substitutes into a correct expression at least once.	M1
		Correct numerical expression (unsimplified)	A1
	= 1 589 463	cao	A1
			(3)
			(11 marks)

Question Number	Scheme	Notes	Marks
7(a)	$u_2 = 3,\ u_3 = 7$		B1, B1
			(2)
(b)	At $n = 1,\ u_1 = 2^1 - 1 = 1$ and so result true for $n = 1$		B1
	Assume true for $n = k$; $u_k = 2^k - 1$		
	and so $u_{k+1}\left(= 2u_k + 1\right) = 2(2^k - 1) + 1$	Substitutes u_k into u_{k+1} (**must see this line**)	M1
		Correct expression	A1
	$u_{k+1}\left(= 2^{k+1} - 2 + 1\right) = 2^{k+1} - 1$	Correct completion to $u_{k+1} = 2^{k+1} - 1$	A1
	Must see 4 things: <u>true for n = 1</u>, <u>assumption true for n = k</u>, <u>said true for n = k + 1</u> and therefore <u>true for all n</u>	Fully complete proof with no errors and comment. All the previous marks in (b) must have been scored.	A1cso
	Ignore any subsequent attempts e.g. $u_{k+2} = 2u_{k+1} + 1 = 2(2^{k+1} - 1) + 1$ **etc.**		(5)
			Total 7

Question Number	Scheme	Notes	Marks
8(a)	$\det(\mathbf{A}) = 3\times 0 - 2\times 1 (= -2)$	Correct attempt at the determinant	M1
	$\det(\mathbf{A}) \neq 0$ (so **A** is non singular)	det(A) = -2 **and** some reference to zero	A1
	$\dfrac{1}{\det(\mathbf{A})}$ **scores M0**		**(2)**
(b)	$\mathbf{BA}^2 = \mathbf{A} \Rightarrow \mathbf{BA} = \mathbf{I} \Rightarrow \mathbf{B} = \mathbf{A}^{-1}$	Recognising that $\mathbf{A}^{-1}$ is required	M1
	$\mathbf{B} = -\dfrac{1}{2}\begin{pmatrix} 3 & -1 \\ -2 & 0 \end{pmatrix}$	At least 3 correct terms in $\begin{pmatrix} 3 & -1 \\ -2 & 0 \end{pmatrix}$	M1
		$\dfrac{1}{\text{their}\det(\text{A})}\begin{pmatrix} * & * \\ * & * \end{pmatrix}$	B1ft
		Fully correct answer	A1
	Correct answer only score 4/4		**(4)**
	Ignore poor matrix algebra notation if the intention is clear		**(6 marks)**

Question Number	Scheme	Notes	Marks
9 (a)	$y = 9x^{-1} \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = -9x^{-2}$ $xy = 9 \Rightarrow x\dfrac{\mathrm{d}y}{\mathrm{d}x} + y = 0$ or $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{dy}{dt}.\dfrac{dt}{dx} = \dfrac{-3}{p^2}.\dfrac{1}{3}$	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = k\,x^{-2}$ Correct use of product rule. The sum of two terms, one of which is correct. their $\dfrac{dy}{dt} \times \left(\dfrac{1}{\text{their } \dfrac{dx}{dt}}\right)$	M1
	$\dfrac{dy}{dx} = -9x^{-2}$ *or* $x\dfrac{dy}{dx} + y = 0$ *or* $\dfrac{dy}{dx} = \dfrac{-3}{p^2}.\dfrac{1}{3}$	Correct differentiation.	A1
	$y - \dfrac{3}{p} = -\dfrac{1}{p^2}(x - 3p)$	Applies $y - \dfrac{3}{p} = (\text{their } m)(x - 3p)$ or $y = (\text{their } m)x + c$ using $x = 3p$ *and* $y = \dfrac{3}{p}$ in an attempt to find c. **Their *m* must be a function of *p* and come from their dy/dx.**	M1
	$x + p^2y = 6p$ *	Cso **given answer**	A1
	Special case – if the correct gradient is <u>quoted</u> could score M0A0M1A1		**(4)**
(b)	$x + q^2y = 6q$	Allow this to score here or in (c)	B1
			(1)
(c)	$6p - p^2y = 6q - q^2y$	Attempt to obtain an **equation** in one variable *x* or *y*	M1
	$y(q^2 - p^2) = 6(q - p) \Rightarrow y = \dfrac{6(q-p)}{q^2 - p^2}$ $x(q^2 - p^2) = 6pq(q - p) \Rightarrow x = \dfrac{6pq(q-p)}{q^2 - p^2}$	Attempt to isolate *x* or *y* – must reach *x* or *y* = f(*p*, *q*) or f(*p*) or f(*q*)	M1
	$y = \dfrac{6}{p+q}$	One correct **simplified** coordinate	A1
	$x = \dfrac{6pq}{p+q}$	Both coordinates correct and **simplified**	A1
			(4)
			(9 marks)